

The background of the book cover is a dramatic photograph of a stormy sea. The water is dark blue and green, with numerous white-capped waves and ripples. Above the horizon, a dark, heavy sky is filled with large, billowing clouds, some illuminated by a bright light source, possibly the moon or the sun, creating a dramatic contrast.

# The Equations of Oceanic Motions

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CAMBRIDGE

# THE EQUATIONS OF OCEANIC MOTIONS

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