



World Survey of Climatology Volume 1B

General Climatology, 1B

Elements of Statistical Analysis

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