McGRAW-HILL

SERIES IN

MECHANICAL

ENGINEERING

G 10

# TURBULENCE

## *An Introduction to Its Mechanism and Theory*

J. O. Hinze

PROFESSOR OF FLUID MECHANICS
TECHNOLOGICAL UNIVERSITY, DELFT

McGRAW-HILL BOOK COMPANY, INC.

New York Toronto London

1959

# CONTENTS

*Preface* . . . . . v

**Chapter 1. General Introduction and Concepts** . . . . . 1

1-1. Definition of Turbulence and Introductory Concepts . . . . . 1
1-2. Equations of Motion for Turbulent Flow; Reynolds Stresses . . . . 13
1-3. Equation for the Conservation of a Transferable Scalar Quantity in a Turbulent Flow . . . . . 24
1-4. Double Correlations between Turbulence-velocity Components . . . 26
1-5. Change in Double Velocity Correlations with Time; Introduction of Triple Velocity Correlations . . . . . 30
1-6. Features of the Double Longitudinal and Lateral Correlations in a Homogeneous Turbulence . . . . . 34
1-7. Macro or Integral Scale of Turbulence . . . . . 37
1-8. Eulerian Correlations with Respect to Time . . . . . 39
1-9. Turbulent Diffusion of Fluid Particles; Lagrangian Correlations . . . 42
1-10. Recapitulation of Correlations . . . . . 49
1-11. Empirical Formulas for Double-correlation Curves . . . . . 51
1-12. Taylor's One-dimensional Energy Spectrum . . . . . 54
1-13. Energy Relations in Turbulent Flows. . . . . 62

**Chapter 2. Principles of Methods and Techniques in the Measurement of Turbulent Flows** . . . . . 73

2-1. Introduction . . . . . 73
2-2. The Hot-wire Anemometer . . . . . 75
2-3. Constant-current Method . . . . . 81
2-4. Measurement of Turbulence Characteristics with the Hot-wire Anemometer; Constant-current Method. . . . . 102
2-5. Measurement of Temperature and Concentration Fluctuations with the Hot-wire Anemometer; Constant-current Method . . . . . 112
2-6. Constant-temperature Method. . . . . 116
2-7. Limitations of the Hot-wire Anemometer. . . . . 119
2-8. Electric-discharge Anemometer. . . . . 122
2-9. The Method of Electromagnetic Induction . . . . . 125
2-10. Methods Based upon Flow Visualization . . . . . 126
2-11. Measurement of Mean Values of Static Pressure and Velocity. . . . 132

**Chapter 3. Isotropic Turbulence**. . . . . 142

3-1. Introduction . . . . . 142
3-2. Correlation Tensors . . . . . 143
3-3. Differential Equation for the Dynamic Behavior of an Isotropic Turbulence . . . . . 158

3-4. The Three-dimensional Energy Spectrum . . . . . 165
3-5. The Dynamic Equation for the Energy Spectrum . . . . . 174
3-6. The Decay of an Isotropic Turbulence . . . . . 204
3-7. Extension to an Isotropic-turbulent Scalar Field . . . . . 219
3-8. Pressure Fluctuations in Isotropic Turbulence . . . . . 239
3-9. Some Remarks on the Effect of Compressibility . . . . . 243

**Chapter 4. Nonisotropic Turbulence** . . . . . 249

4-1. Introduction . . . . . 249
4-2. Dynamics of the One-point Velocity Correlation $\overline{u_i u_j}$ . . . . . 250
4-3. Dynamics of the Two-point Velocity Correlation $\overline{(u_i)_A (u_j)_B}$ . . . . 256
4-4. The Dynamic Equation for the Energy Spectrum . . . . . 260

**Chapter 5. Transport Processes in Turbulent Flows** . . . . . 275

5-1. Introduction . . . . . 275
5-2. Mixing-length and Phenomenological Theories . . . . . 277
5-3. Analogies in Turbulence Transport . . . . . 293
5-4. Diffusion in Homogeneous Turbulence . . . . . 302
5-5. Diffusion from a Fixed Source in a Uniform Flow . . . . . 324
5-6. Diffusion from a Fixed Source in a Turbulent Shear Flow . . . . . 342
5-7. Diffusion of Discrete Particles in a Homogeneous Turbulence . . . . 352
5-8. Effect of Compressibility . . . . . 365

**Chapter 6. Nonisotropic Free Turbulence** . . . . . 376

6-1. Introduction . . . . . 376
6-2. Approximations Applied to the Equations of Motion . . . . . 378
6-3. Velocity Distribution Behind a Cylinder According to the Classical Theories . . . . . 384
6-4. The Transport of a Scalar Quantity in the Wake Flow of a Cylinder . . 390
6-5. Measurements of Mean-velocity and Mean-temperature Distribution in the Wake Flow of a Cylinder . . . . . 393
6-6. Measurements of Turbulence Quantities in the Wake Flow of a Cylinder 396
6-7. Velocity Distribution in a Round Free Jet According to the Classical Theories . . . . . 404
6-8. The Transport of a Scalar Quantity in a Round Free Jet . . . . . 416
6-9. Measurements of Mean-velocity and Mean-temperature Distribution in a Round Free Jet . . . . . 420
6-10. Measurements of Turbulence Quantities in a Round Free Jet . . . . 431
6-11. The Structure of Free Turbulent Shear Flow and Transport Processes. 439

**Chapter 7. Nonisotropic "Wall" Turbulence** . . . . . 451

7-1. Introduction . . . . . 451
7-2. Approximations to the Equations of Motion, and Their Integral Relations 453
7-3. The Laminar Boundary Layer and Transition . . . . . 462
7-4. Turbulent Boundary Layer Along a Flat Plate. Classical Theories . . 465
7-5. Measurements on Mean-velocity Distribution . . . . . 476
7-6. Measurements on Turbulence Quantities in a Boundary Layer . . . 487
7-7. Recapitulation and New Expressions for Mean-velocity and Shear-stress Distribution . . . . . 503
7-8. Turbulent Flow through a Straight Circular Pipe. Mean-velocity Distribution . . . . . 514

7-9. Measurements of Turbulence Quantities in Pipe Flow . . . . . 520
7-10. The Structure of Turbulent Pipe Flow . . . . . 533
7-11. Transport of a Scalar Quantity in Wall Turbulence . . . . . 537
7-12. Miscellaneous Problems in Wall Turbulence . . . . . 553

*Appendix.* Elements of Cartesian Tensors . . . . . 567

*Name Index* . . . . . 577

*Subject Index* . . . . . 581