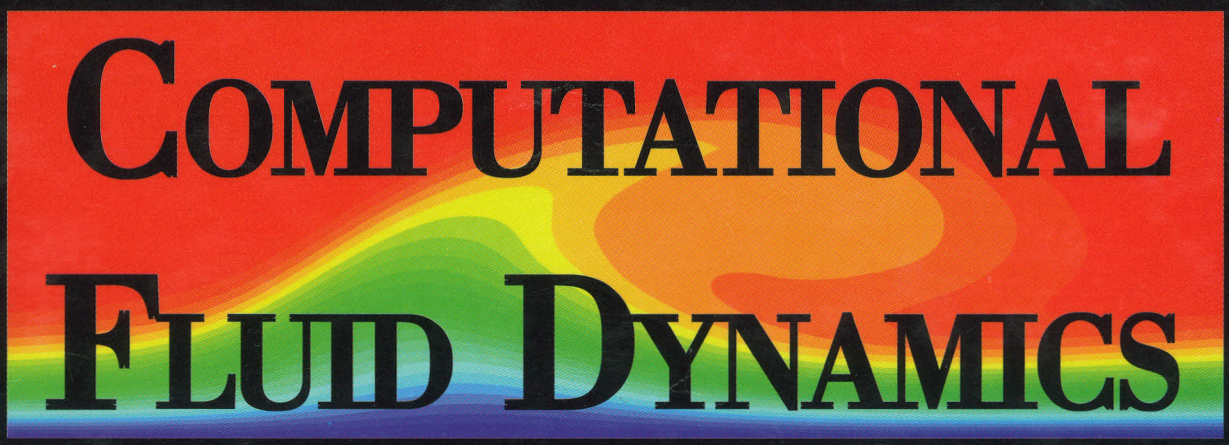
FUNDAMENTALS OF COMPUTATIONAL FLUID DYNAMICS



PATRICK J. ROACHE

FUNDAMENTALS OF COMPUTATIONAL FLUID DYNAMICS

BY

PATRICK J. ROACHE

4416/376
Leibniz Universität Hannover
Institut für
Meteorologie und Klimatologie
Herrenhäuser Str. 2 · 30419 Hannover

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