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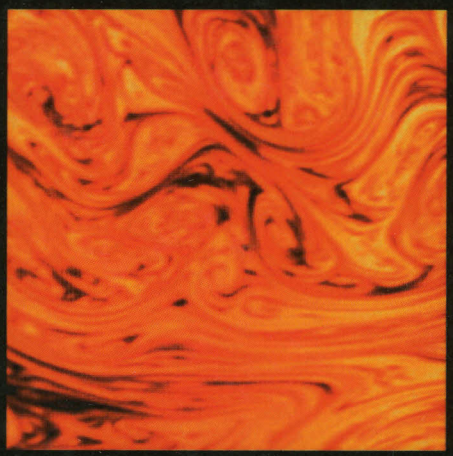
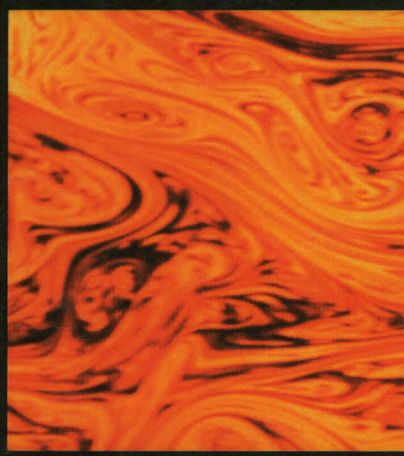

turbulence

AN INTRODUCTION FOR SCIENTISTS AND ENGINEERS

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Turbulence

An Introduction for Scientists and Engineers

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