Jie-Zhi Wu
Hui-Yang Ma
Ming-De Zhou

Vorticity and Vortex Dynamics

Springer

J.-Z. Wu H.-Y. Ma M.-D. Zhou

Vorticity and Vortex Dynamics

With 291 Figures

D III 293

DK: 551.511.2 551.511.6
551.511.3 551.511.61
551.511.32

358/4271 INSTITUT
FÜR METEOROLOGIE U. KLIMATOLOGIE
UNIVERSITÄT HANNOVER
HERRENHÄUSER STR. 2 · 30419 HANNOVER

Springer

Contents

1 Introduction 1
1.1 Fundamental Processes in Fluid Dynamics and Their Coupling 2
1.2 Historical Development 3
1.3 The Contents of the Book 6

Part I Vorticity Dynamics

2 Fundamental Processes in Fluid Motion 13
2.1 Basic Kinematics 13
2.1.1 Descriptions and Visualizations of Fluid Motion 13
2.1.2 Deformation Kinematics. Vorticity and Dilatation 18
2.1.3 The Rate of Change of Material Integrals 22
2.2 Fundamental Equations of Newtonian Fluid Motion 25
2.2.1 Mass Conservation 25
2.2.2 Balance of Momentum and Angular Momentum 26
2.2.3 Energy Balance, Dissipation, and Entropy 28
2.2.4 Boundary Conditions. Fluid-Dynamic Force and Moment 30
2.2.5 Effectively Inviscid Flow and Surface of Discontinuity 33
2.3 Intrinsic Decompositions of Vector Fields 36
2.3.1 Functionally Orthogonal Decomposition 36
2.3.2 Integral Expression of Decomposed Vector Fields 40
2.3.3 Monge–Clebsch decomposition 43
2.3.4 Helical–Wave Decomposition 44
2.3.5 Tensor Potentials 47
2.4 Splitting and Coupling of Fundamental Processes 48
2.4.1 Triple Decomposition of Strain Rate and Velocity Gradient 49

2.4.2 Triple Decomposition of Stress Tensor and Dissipation ... 52
2.4.3 Internal and Boundary Coupling of Fundamental Processes ... 55
2.4.4 Incompressible Potential Flow ... 59
Summary ... 63

3 Vorticity Kinematics ... 67
3.1 Physical Interpretation of Vorticity ... 67
3.2 Vorticity Integrals and Far-Field Asymptotics ... 71
3.2.1 Integral Theorems ... 71
3.2.2 Biot–Savart Formula ... 78
3.2.3 Far-Field Velocity Asymptotics ... 83
3.3 Lamb Vector and Helicity ... 85
3.3.1 Complex Lamellar, Beltrami, and Generalized Beltrami Flows ... 86
3.3.2 Lamb Vector Integrals, Helicity, and Vortex Filament Topology ... 90
3.4 Vortical Impulse and Kinetic Energy ... 94
3.4.1 Vortical Impulse and Angular Impulse ... 94
3.4.2 Hydrodynamic Kinetic Energy ... 97
3.5 Vorticity Evolution ... 100
3.5.1 Vorticity Evolution in Physical and Reference Spaces ... 100
3.5.2 Evolution of Vorticity Integrals ... 103
3.5.3 Enstrophy and Vorticity Line Stretching ... 105
3.6 Circulation-Preserving Flows ... 109
3.6.1 Local and Integral Conservation Theorems ... 109
3.6.2 Bernoulli Integrals ... 113
3.6.3 Hamiltonian Formalism ... 117
3.6.4 Relabeling Symmetry and Energy Extremum ... 120
3.6.5 Viscous Circulation-Preserving Flow ... 125
Summary ... 127

4 Fundamentals of Vorticity Dynamics ... 131
4.1 Vorticity Diffusion Vector ... 131
4.1.1 Nonconservative Body Force in Magnetohydrodynamics ... 131
4.1.2 Baroclinicity ... 134
4.1.3 Viscosity Diffusion, Dissipation, and Creation at Boundaries ... 138
4.1.4 Unidirectional and Quasiparallel Shear Flows ... 144

4.2 Vorticity Field at Small Reynolds Numbers 150
4.2.1 Stokes Approximation of Flow Over Sphere 150
4.2.2 Oseen Approximation of Flow Over Sphere 153
4.2.3 Separated Vortex and Vortical Wake 155
4.2.4 Regular Perturbation 159
4.3 Vorticity Dynamics in Boundary Layers 161
4.3.1 Vorticity and Lamb Vector in Solid-Wall Boundary Layer 162
4.3.2 Vorticity Dynamics in Free-Surface Boundary Layer 168
4.4 Vortex Sheet Dynamics 172
4.4.1 Basic Properties 173
4.4.2 Kutta Condition 178
4.4.3 Self-Induced Motion 179
4.4.4 Vortex Sheet Transport Equation 183
4.5 Vorticity-Based Formulation of Viscous Flow Problem 185
4.5.1 Kinematical Well-Posedness 187
4.5.2 Boundary Vorticity–Pressure Coupling 190
4.5.3 A Locally Decoupled Differential Formulation 191
4.5.4 An Exact Fully Decoupled Formulation 197
Summary 199

5 Vorticity Dynamics in Flow Separation 201
5.1 Flow Separation and Boundary-Layer Separation 201
5.2 Three-Dimensional Steady Flow Separation 204
5.2.1 Near-Wall Flow in Terms of On-Wall Signatures 205
5.2.2 Local Separation Criteria 210
5.2.3 Slope of Separation Stream Surface 213
5.2.4 A Special Result on Curved Surface 215
5.3 Steady Boundary Layer Separation 216
5.3.1 Goldstein's Singularity and Triple-Deck Structure 218
5.3.2 Triple-Deck Equations and Interactive Vorticity Generation 221
5.3.3 Boundary-Layer Separation in Two Dimensions 227
5.3.4 Boundary-Layer Separation in Three Dimensions 229
5.4 Unsteady Separation 234
5.4.1 Physical Phenomena of Unsteady Boundary-Layer Separation 235
5.4.2 Lagrangian Theory of Unsteady Boundary Layer Separation 240
5.4.3 Unsteady Flow Separation 246
Summary 251

Part II Vortex Dynamics

6 Typical Vortex Solutions 255
6.1 Governing Equations 255
6.2 Axisymmetric Columnar Vortices 260
6.2.1 Stretch-Free Columnar Vortices 260
6.2.2 Viscous Vortices with Axial Stretching 263
6.2.3 Conical Similarity Swirling Vortices 268
6.3 Circular Vortex Rings 272
6.3.1 General Formulation and Induced Velocity 272
6.3.2 Fraenkel–Norbury Family and Hill Spherical Vortex 277
6.3.3 Thin-Cored Pure Vortex Ring: Direct Method 281
6.3.4 Thin-Cored Swirling Vortex Rings: Energy Method 283
6.4 Exact Strained Vortex Solutions 284
6.4.1 Strained Elliptic Vortex Patches 285
6.4.2 Vortex Dipoles 289
6.4.3 Vortex Arrays 291
6.5 Asymptotic Strained Vortex Solutions 295
6.5.1 Matched Asymptotic Expansion and Canonical Equations 296
6.5.2 Strained Solution in Distant Vortex Dipole 303
6.5.3 Vortex in Triaxial Strain Field 306
6.6 On the Definition of Vortex 310
6.6.1 Existing Criteria 310
6.6.2 An Analytical Comparison of the Criteria 314
6.6.3 Test Examples and Discussion 316
Summary 320

7 Separated Vortex Flows 323
7.1 Topological Theory of Separated Flows 323
7.1.1 Fixed Points and Closed Orbits of a Dynamic System 324
7.1.2 Closed and Open Separations 327
7.1.3 Fixed-Point Index and Topology of Separated Flows 330
7.1.4 Structural Stability and Bifurcation of Separated Flows 332
7.2 Steady Separated Bubble Flows in Euler Limit 339
7.2.1 Prandtl–Batchelor Theorem 340
7.2.2 Plane Prandtl–Batchelor Flows 346
7.2.3 Steady Global Wake in Euler Limit 350
7.3 Steady Free Vortex-Layer Separated Flow 352
7.3.1 Slender Approximation of Free Vortex Sheet 353

7.3.2 Vortex Sheets Shed from Slender Wing 359
7.3.3 Stability of Vortex Pairs Over Slender Conical Body . . . 361
7.4 Unsteady Bluff-Body Separated Flow 366
7.4.1 Basic Flow Phenomena 367
7.4.2 Formation of Vortex Shedding 372
7.4.3 A Dynamic Model of the (St, C_D, Re) Relationship 376
Summary 381

8 Core Structure, Vortex Filament, and Vortex System 383
8.1 Vortex Formation and Core Structure 383
8.1.1 Vortex Formation by Vortex-Layer Rolling Up 384
8.1.2 Quasicylindrical Vortex Core 387
8.1.3 Core Structure of Typical Vortices 390
8.1.4 Vortex Core Dynamics 395
8.2 Dynamics of Three-Dimensional Vortex Filament 399
8.2.1 Local Induction Approximation 401
8.2.2 Vortex Filament with Finite Core and Stretching 407
8.2.3 Nonlocal Effects of Self-Stretch and Background Flow 413
8.3 Motion and Interaction of Multiple Vortices 418
8.3.1 Two-Dimensional Point-Vortex System 418
8.3.2 Vortex Patches 424
8.3.3 Vortex Reconnection 431
8.4 Vortex–Boundary Interactions 434
8.4.1 Interaction of Vortex with a Body 435
8.4.2 Interaction of Vortex with Fluid Interface 441
Summary 446

Part III Vortical Flow Instability, Transition and Turbulence

9 Vortical-Flow Stability and Vortex Breakdown 451
9.1 Fundamentals of Hydrodynamic Stability 451
9.1.1 Normal-Mode Linear Stability 453
9.1.2 Linear Instability with Non-normal Operator 458
9.1.3 Energy Method and Inviscid Arnold Theory 462
9.1.4 Linearized Disturbance Lamb Vector and the Physics of Instability 467
9.2 Shear-Flow Instability 469
9.2.1 Instability of Parallel Shear Flow 469
9.2.2 Instability of free shear flow 472
9.2.3 Instability of Boundary Layer 475
9.2.4 Non-Normal Effects in Shear-Flow Instability 477

9.3 Instability of Axisymmetric Columnar Vortices 480
9.3.1 Stability of Pure Vortices 480
9.3.2 Temporal Instability of Swirling Flow 481
9.3.3 Absolute and Convective Instability of Swirling Flow 485
9.3.4 Non-Modal Instability of Vortices 488
9.4 Instabilities of Strained Vortices 492
9.4.1 Elliptical Instability 493
9.4.2 A Columnar Vortex in a Strained Field 496
9.4.3 Instability of a Vortex Pair 499
9.5 Vortex Breakdown 502
9.5.1 Vorticity-Dynamics Mechanisms of Vortex Breakdown 504
9.5.2 Onset of Vortex Breakdown: Fold Catastrophe Theory 506
9.5.3 Vortex Breakdown Development: AI/CI Analysis 511
Summary 515

10 Vortical Structures in Transitional and Turbulent Shear Flows 519
10.1 Coherent Structures 520
10.1.1 Coherent Structures and Vortices 520
10.1.2 Scaling Problem in Coherent Structure 522
10.1.3 Coherent Structure and Wave 524
10.2 Vortical Structures in Free Shear Flows 526
10.2.1 Instability of Free Shear Layers and Formation of Spanwise Vortices 526
10.2.2 The Secondary Instability and Formation of Streamwise Vortices 530
10.2.3 Vortex Interaction and Small-Scale Transition 532
10.3 Vortical Structures in Wall-Bounded Shear Layers 535
10.3.1 Tollmien–Schlichting Instability and Formation of Initial Streaks 536
10.3.2 Secondary Instability and Self-Sustaining Cycle of Structure Regeneration 539
10.3.3 Small-Scale Transition in Boundary Layers 541
10.3.4 A General Description of Turbulent Boundary Layer Structures 545
10.3.5 Streamwise Vortices and By-Pass Transition 548
10.4 Some Theoretical Aspects in Studying Coherent Structures ... 550
10.4.1 On the Reynolds Decomposition 551
10.4.2 On Vorticity Transport Equations 556
10.4.3 Vortex Core Dynamics and Polarized Vorticity Dynamics 559

10.5 Two Basic Processes in Turbulence 561
10.5.1 Coherence Production – the First Process 562
10.5.2 Cascading – the Second Process 566
10.5.3 Flow Chart of Coherent Energy and General Strategy of Turbulence Control 567
10.6 Vortical Structures in Other Shear Flows 573
10.6.1 Vortical Structures in Plane Complex Turbulent Shear Flows 573
10.6.2 Vortical Structures in Nonplanar Shear Flows 577
10.6.3 Vortical Flow Shed from Bluff Bodies 580
Summary 583

Part IV Special Topics

11 Vortical Aerodynamic Force and Moment 587
11.1 Introduction 587
11.1.1 The Need for "Nonstandard" Theories 588
11.1.2 The Legacy of Pioneering Aerodynamicist 590
11.1.3 Exact Integral Theories with Local Dynamics 593
11.2 Projection Theory 594
11.2.1 General Formulation 595
11.2.2 Diagnosis of Pressure Force Constituents 597
11.3 Vorticity Moments and Classic Aerodynamics 599
11.3.1 General Formulation 600
11.3.2 Force, Moment, and Vortex Loop Evolution 603
11.3.3 Force and Moment on Unsteady Lifting Surface 606
11.4 Boundary Vorticity-Flux Theory 608
11.4.1 General Formulation 608
11.4.2 Airfoil Flow Diagnosis 611
11.4.3 Wing-Body Combination Flow Diagnosis 615
11.5 A DMT-Based Arbitrary-Domain Theory 617
11.5.1 General Formulation 617
11.5.2 Multiple Mechanisms Behind Aerodynamic Forces 621
11.5.3 Vortex Force and Wake Integrals in Steady Flow 627
11.5.4 Further Applications 633
Summary 639

12 Vorticity and Vortices in Geophysical Flows 641
12.1 Governing Equations and Approximations 642
12.1.1 Effects of Frame Rotation and Density Stratification 642
12.1.2 Boussinesq Approximation 646
12.1.3 The Taylor–Proudman Theorem 648
12.1.4 Shallow-Water Approximation 649

12.2 Potential Vorticity 652
12.2.1 Barotropic (Rossby) Potential Vorticity 653
12.2.2 Geostrophic and Quasigeostrophic Flows 654
12.2.3 Rossby Wave 656
12.2.4 Baroclinic (Ertel) Potential Vorticity 659
12.3 Quasigeostrophic Evolution of Vorticity and Vortices 664
12.3.1 The Evolution of Two-Dimensional Vorticity Gradient 665
12.3.2 The Structure and Evolution of Barotropic Vortices 670
12.3.3 The Structure of Baroclinic Vortices 676
12.3.4 The Propagation of Tropical Cyclones 680
Summary 690

A Vectors, Tensors, and Their Operations 693
A.1 Vectors and Tensors 693
A.1.1 Scalars and Vectors 693
A.1.2 Tensors 694
A.1.3 Unit Tensor and Permutation Tensor 696
A.2 Integral Theorems and Derivative Moment Transformation 698
A.2.1 Generalized Gauss Theorem and Stokes Theorem 698
A.2.2 Derivative Moment Transformation on Volume 700
A.2.3 Derivative Moment Transformation on Surface 701
A.2.4 Special Issues in Two Dimensions 703
A.3 Curvilinear Frames on Lines and Surfaces 705
A.3.1 Intrinsic Line Frame 705
A.3.2 Intrinsic operation with surface frame 707
A.4 Applications in Lagrangian Description 716
A.4.1 Deformation Gradient Tensor and its Inverse 716
A.4.2 Images of Physical Vectors in Reference Space 717

References 721

Index 767