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Theory and Computation in Hydrodynamic Stability

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THEORY AND COMPUTATION OF HYDRODYNAMIC STABILITY

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