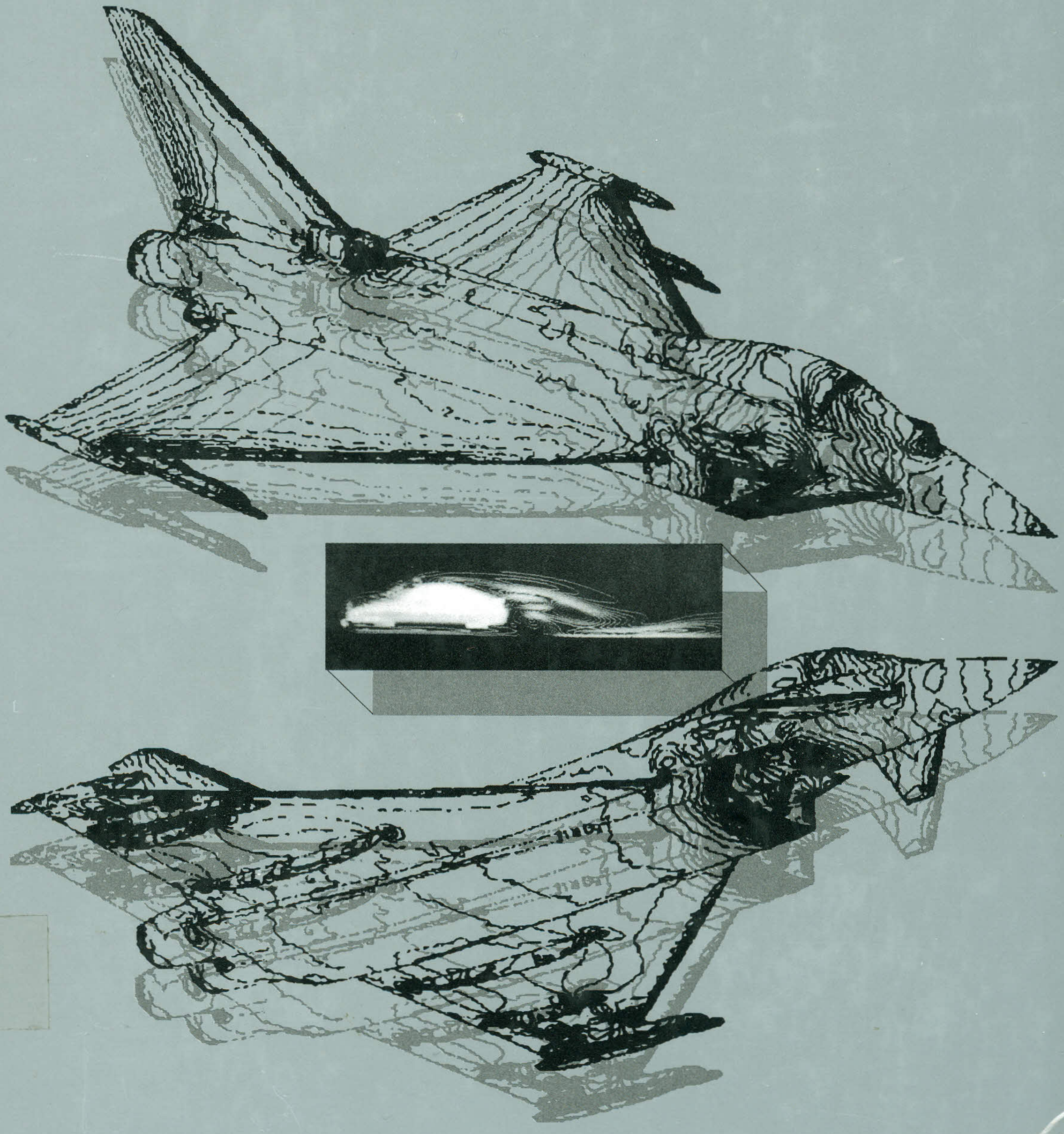
JOHN D. ANDERSON, JR.

Computational Fluid Dynamics

THE BASICS WITH APPLICATIONS



COMPUTATIONAL FLUID DYNAMICS

The Basics with Applications

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