

Pierre Sagaut

Large Eddy Simulation for Incompressible Flows

Scientific
Computation

An Introduction



Springer

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An Introduction

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