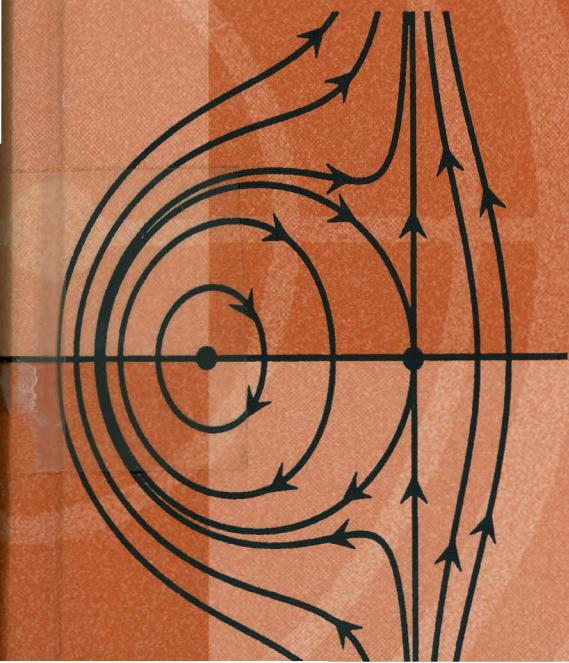


Fritz K. Kneubühl

# Oscillations and Waves



Springer

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With 146 Figures and 16 Tables

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