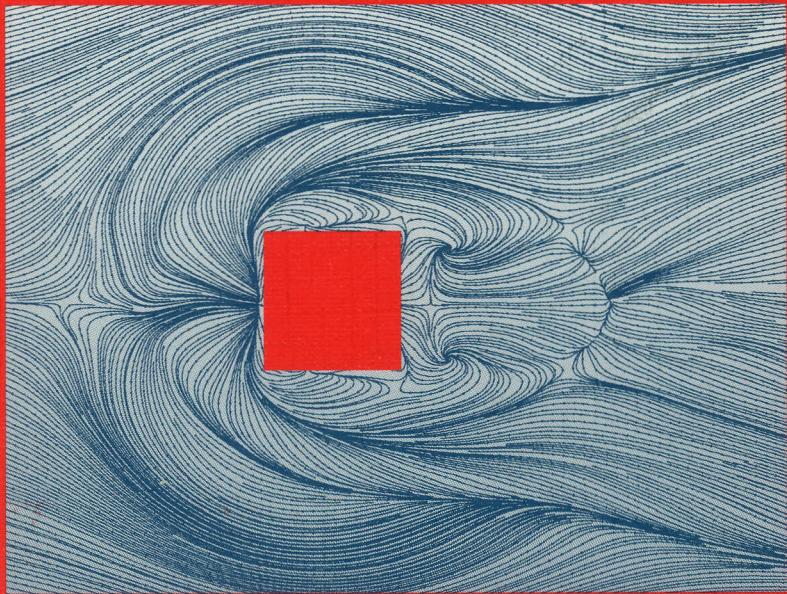


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Computational Methods for Fluid Dynamics



Springer

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With 95 Figures

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