

Alfred K. Blackadar

# Turbulence and Diffusion in the Atmosphere

Lectures  
in Environmental  
Sciences



Springer

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# Turbulence and Diffusion in the Atmosphere

Lectures  
in Environmental Sciences

With 49 Figures  
and Two MS-DOS Program Diskettes

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FÜR METEOROLOGIE U. KLIMATOLOGIE  
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