

Handwritten text in a cursive script, likely a historical document or manuscript. The text is arranged in several lines, with some words appearing to be in a different script or dialect. The ink is dark and the paper shows signs of age.



Handwritten text in a cursive script, located below the sketch of the face. The text is arranged in several lines, with some words appearing to be in a different script or dialect. The ink is dark and the paper shows signs of age.

Handwritten text in a cursive script, located below the first block of text. The text is arranged in several lines, with some words appearing to be in a different script or dialect. The ink is dark and the paper shows signs of age.



TURBULENCE

Uriel Frisch

DIII 238

DK 551.511.6

TURBULENCE

THE LEGACY OF A. N. KOLMOGOROV

URIEL FRISCH

Observatoire de la Côte d'Azur

310/3877 INSTITUT
FÜR METEOROLOGIE U. KLIMATOLOGIE
UNIVERSITÄT HANNOVER
HERRENHAUSER STR. 2 - 30119 HANNOVER



CAMBRIDGE
UNIVERSITY PRESS

Contents

Preface *page xi*

CHAPTER 1

Introduction

1.1	Turbulence and symmetries	1
1.2	Outline of the book	11

CHAPTER 2

Symmetries and conservation laws

2.1	Periodic boundary conditions	14
2.2	Symmetries	17
2.3	Conservation laws	18
2.4	Energy budget scale-by-scale	21

CHAPTER 3

Why a probabilistic description of turbulence?

3.1	There is something predictable in a turbulent signal	27
3.2	A model for deterministic chaos	31
3.3	Dynamical systems	36
3.4	The Navier–Stokes equation as a dynamical system	37

CHAPTER 4

Probabilistic tools: a survey

4.1	Random variables	40
4.2	Random functions	45
4.3	Statistical symmetries	46
4.4	Ergodic results	49
4.5	The spectrum of stationary random functions	52

CHAPTER 5

Two experimental laws of fully developed turbulence

5.1	The two-thirds law	57
5.2	The energy dissipation law	67

CHAPTER 6

The Kolmogorov 1941 theory

6.1	Kolmogorov 1941 and symmetries	72
6.2	Kolmogorov's four-fifths law	76
6.2.1	The Kármán–Howarth–Monin relation for anisotropic turbulence	77
6.2.2	The energy flux for homogeneous turbulence.....	79
6.2.3	The energy flux for homogeneous isotropic turbulence.	81
6.2.4	From the energy flux relation to the four-fifths law	84
6.2.5	Remarks on Kolmogorov's four-fifths law	86
6.3	Main results of the Kolmogorov 1941 theory	89
6.3.1	The Kolmogorov–Obukhov law and the structure functions	89
6.3.2	Effect of a finite viscosity: the dissipation range	91
6.4	Kolmogorov and Landau: the lack of universality	93
6.4.1	The original formulation of Landau's objection	93
6.4.2	A modern reformulation of Landau's objection	94
6.4.3	Kolmogorov and Landau reconciled?.....	97
6.5	Historical remarks on the Kolmogorov 1941 theory	98

CHAPTER 7

Phenomenology of turbulence in the sense of Kolmogorov 1941

7.1	Introduction	100
7.2	Basic tools of phenomenology	101
7.3	The Richardson cascade and the localness of interactions	103
7.4	Reynolds numbers and degrees of freedom	106
7.5	Microscopic and macroscopic degrees of freedom	109
7.6	The distribution of velocity gradients	111
7.7	The law of decay of the energy	112
7.8	Beyond phenomenology: finite-time blow-up of ideal flow	115

CHAPTER 8

Intermittency

8.1	Introduction	120
8.2	Self-similar and intermittent random functions	121
8.3	Experimental results on intermittency	127
8.4	Exact results on intermittency	133
8.5	Intermittency models based on the velocity	135
8.5.1	The β -model	135
8.5.2	The bifractal model	140
8.5.3	The multifractal model	143
8.5.4	A probabilistic reformulation of the multifractal model	146
8.5.5	The intermediate dissipation range and multifractal universality	149
8.5.6	The skewness and the flatness of velocity derivatives according to the multifractal model	155
8.6	Intermittency models based on the dissipation	159
8.6.1	Multifractal dissipation	159
8.6.2	Bridging multifractality based on the velocity and multifractality based on the dissipation	163
8.6.3	Random cascade models	165
8.6.4	Large deviations and multifractality	168
8.6.5	The lognormal model and its shortcomings	171

8.7	Shell models	174
8.8	Historical remarks on fractal intermittency models	178
8.9	Trends in intermittency research	182
8.9.1	Vortex filaments: the sinews of turbulence?	185
8.9.2	Statistical signature of vortex filaments: dog or tail?... ..	188
8.9.3	The distribution of velocity increments	192

CHAPTER 9

Further reading: a guided tour

9.1	Introduction	195
9.2	Books on turbulence and fluid mechanics	195
9.3	Mathematical aspects of fully developed turbulence	199
9.4	Dynamical systems, fractals and turbulence	203
9.5	Closure, functional and diagrammatic methods	206
9.5.1	The Hopf equation	207
9.5.2	Functional and diagrammatic methods	212
9.5.3	The direct interaction approximation	217
9.5.4	Closures and their shortcomings	219
9.6	Eddy viscosity, multiscale methods and renormalization	222
9.6.1	Eddy viscosity: a very old idea	222
9.6.2	Multiscale methods	226
9.6.3	Applications of multiscale methods in turbulence	230
9.6.4	Renormalization group (RG) methods	235
9.7	Two-dimensional turbulence	240
9.7.1	Cascades and vortices	241
9.7.2	Two-dimensional turbulence and statistical mechanics .	243
9.7.3	Conservative dynamics 'punctuated' by dissipative events	249
9.7.4	From Flatland to three-dimensional turbulence	251
	<i>References</i>	255
	<i>Author index</i>	283
	<i>Subject index</i>	289