

Alexandre J. Chorin

**Applied
Mathematical
Sciences
103**

Vorticity and Turbulence



Springer-Verlag

Alexandre J. Chorin

Vorticity and Turbulence

With 45 Illustrations



Springer-Verlag

New York Berlin Heidelberg London Paris
Tokyo Hong Kong Barcelona Budapest

Contents

Preface	v
Introduction	1
1. The Equations of Motion	5
1.1. The Euler and Navier-Stokes Equations	5
1.2. Vorticity Form of the Equations	9
1.3. Discrete Vortex Representations	12
1.4. Magnetization Variables	17
1.5. Fourier Representation for Periodic Flow	21
2. Random Flow and Its Spectra	25
2.1. Introduction to Probability Theory	25
2.2. Random Fields	30
2.3. Random Solutions of the Navier-Stokes Equations	36
2.4. Random Fourier Transform of a Homogeneous Flow Field	39
2.5. Brownian Motion and Brownian Walks	44
3. The Kolmogorov Theory	49
3.1. The Goals of Turbulence Theory: Universal Equilibrium	49
3.2. Kolmogorov Theory: Dimensional Considerations	51
3.3. The Kolmogorov Spectrum and an Energy Cascade	55
3.4. Fractal Dimension	58
3.5. A First Discussion of Intermittency	61

4. Equilibrium Flow in Spectral Variables and in Two Space Dimensions	67
4.1. Statistical Equilibrium	67
4.2. The "Absolute" Statistical Equilibrium in Wave Number Space	72
4.3. The Combinatorial Method: The Approach to Equilibrium and Negative Temperatures	74
4.4. The Onsager Theory and the Joyce-Montgomery Equation .	77
4.5. The Continuum Limit and the Role of Invariants	80
4.6. The Approach to Equilibrium, Viscosity, and Inertial Power Laws	84
5. Vortex Stretching	91
5.1. Vortex Lines Stretch	91
5.2. Vortex Filaments	94
5.3. Self-Energy and the Folding of Vortex Filaments	96
5.4. Fractalization and Capacity	99
5.5. Intermittency	101
5.6. Vortex Cross-Sections	106
5.7. Enstrophy and Equilibrium	108
6. Polymers, Percolation, Renormalization	113
6.1. Spins, Critical Points and Metropolis Flow	113
6.2. Polymers and the Flory Exponent	116
6.3. The Vector-Vector Correlation Exponent for Polymers . . .	119
6.4. Percolation	121
6.5. Polymers and Percolation	124
6.6. Renormalization	126
6.7. The Kosterlitz-Thouless Transition	128
6.8. Vortex Percolation/ λ Transition in Three Space Dimensions	132
7. Vortex Equilibria in Three-Dimensional Space	135
7.1. A Vortex Filament Model	135
7.2. Self-Avoiding Filaments of Finite Length	137
7.3. The Limit $N \rightarrow \infty$ and the Kolmogorov Exponent	140
7.4. Dynamics of a Vortex Filament: Viscosity and Reconnection	144
7.5. Relation to the λ Transition in Superfluids: Denser Suspensions of Vortices	149
7.6. Renormalization of Vortex Dynamics in a Turbulent Regime	152
Bibliography	157
Index	169