Tan Weiyan

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SHALLOW WATER HYDRODYNAMICS

Mathematical Theory and Numerical Solution for a Two-dimensional System of Shallow Water Equations

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