

THE INSTITUTE OF MATHEMATICS & ITS
APPLICATIONS MONOGRAPH SERIES

**AN INFORMAL
INTRODUCTION
TO THEORETICAL
FLUID MECHANICS**

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An Informal Introduction to Theoretical Fluid Mechanics

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