

NOTES ON
NUMERICAL FLUID MECHANICS

Volume 4

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Shear Flow in
Surface-Oriented Coordinates

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With 40 Figures



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