

V. K. Stokes

# Theories of Fluids with Microstructure

An Introduction



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## An Introduction

With 44 Figures

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