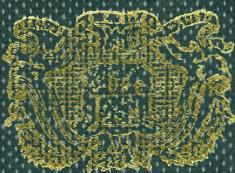


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LXXXVIII Corso

*Turbolenza e predicitività  
nella fluidodinamica geofisica  
e la dinamica del clima*



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