

Nonlinear Dynamics and Turbulence

Edited by

G I Barenblatt

G Iooss

D D Joseph

Interaction of
Mechanics and
Mathematics
Series

Pitman Advanced Publishing Program
BOSTON · LONDON · MELBOURNE



Nonlinear Dynamics and Turbulence

14312454 INSTITUT
FÜR METEOROLOGIE U. KLIMATOLOGIE
UNIVERSITÄT HANNOVER
HERRENHAUSER STR. 2 • 3000 HANNOVER 21

Edited by

G I Barenblatt

P P Shirsov Institute of Oceanology
USSR Academy of Sciences

G Iooss

Institut de Mathématiques et Sciences Physiques
Université de Nice

D D Joseph

Department of Aerospace Engineering and Mechanics
University of Minnesota



Pitman Advanced Publishing Program

BOSTON • LONDON • MELBOURNE

Contents

Foreword v
List of contributors xii
Preface xiii

1 Strange attractors and quasiattractors 1

V S Afraimovich and L P Shil'nikov

1.1 Introduction 1
1.2 Statement of the problem 11
1.3 Structure of a nonwandering set—basic results 17
1.4 Lorenz attractors—lacunae 22
1.5 Quasiattractors 27
References 31

2 Resonance phenomena for two-parameter families of maps of the plane: uniqueness and nonuniqueness of rotation numbers 35

D G Aronson, M. A. Chory, G R Hall and R P McGehee

2.1 Observations on a one-parameter family 35
2.2 Embedding in a two-parameter family 40
2.3 Criteria for uniqueness and nonuniqueness of rotation number 42
Acknowledgements 46
References 46

3 Selfsimilar turbulence propagation from an instantaneous plane source 48

G I Barenblatt

3.1 Statement of the problem 48
3.2 Exact solution for the limiting case of lack of dissipation 51

- 3.3 Solution for the case of a finite dissipation—selfsimilar intermediate asymptotics 52
- 3.4 Effect of stratification 56
- 3.5 Conclusion 59
- References 59

- 4 Transition to stochasticity of viscous flow between rotating spheres 61**
Yu N Belayev and I M Yavorskaya
 - 4.1 Theoretical and experimental background 61
 - 4.2 Spectral and correlation analysis 63
 - 4.3 Comparison with other flows—discussion 68
 - References 69

- 5 Statistical properties of Lorenz attractors 71**
L A Bunimovich
 - 5.1 Introduction—ergodic theory of dissipative systems 71
 - 5.2 The Lorenz system and its simplest properties 73
 - 5.3 Symbolic dynamics for the Lorenz attractor and some auxiliary assertions 77
 - 5.4 Estimation of the correlation function decay 83
 - 5.5 The central limit theorem 86
 - Acknowledgements 90
 - References 91

- 6 An example of direct bifurcation into a turbulent state 93**
F H Busse
 - 6.1 Introduction 93
 - 6.2 Mathematical formulation 94
 - 6.3 Initial value problem 96
 - 6.4 Statistical limit cycle 98
 - 6.5 Discussion 99
 - References 100

- 7 Universal behaviour in nonlinear systems 101**
M J Feigenbaum
 - 7.1 Introduction 101
 - 7.2 Functional iteration 103
 - 7.3 Fixed-point behaviour of functional iterations 105
 - 7.4 Period 2 from the fixed point 107
 - 7.5 Period doubling *ad infinitum* 114
 - 7.6 The universal limit of high iterates 117
 - 7.7 Some details of the full theory 119
 - 7.8 Universal behaviour in higher dimensional systems 128

- 7.9 Universal behaviour in differential systems 129
- 7.10 Onset of turbulence 135
 - References 138

8 Asymptotic numerical analysis for the Navier–Stokes equations 139

C Foias and R Temam

- 8.1 Introduction 139
- 8.2 Notation and recapitulation of results 139
- 8.3 Approximation in the subspaces V_m 142
- 8.4 Approximation in a general subspace 145
- 8.5 Time-periodic solutions 149
- 8.6 Comment on the Galerkin approximation 151
 - References 154

9 Recent experiments on the transition to turbulent convection 156

J P Gollub

- 9.1 Introduction 156
- 9.2 New methods 157
- 9.3 Small aspect ratio convection 158
- 9.4 Large aspect ratio convection 163
- 9.5 Conclusion 169
 - References 169

10 Homoclinic orbits, subharmonics and global bifurcations in forced oscillations 172

B D Greenspan and P J Holmes

- 10.1 Introduction—a physical example 172
- 10.2 Poincaré maps and invariant manifolds 175
- 10.3 Perturbations of integrable systems—Melnikov’s method 178
- 10.4 Global bifurcations of duffing’s equation 188
- 10.5 Smale horseshoes, Newhouse sinks and chaotic motions 193
- 10.6 Global structure of solutions of Duffing’s equation 198
 - Appendix: Invariant manifolds and the lambda Lemma 210
 - Acknowledgement 211
 - References 211

11 A review of interactions of Hopf and steady-state bifurcations 215

W F Langford

- 11.1 Introduction 215
- 11.2 Preliminaries 216
- 11.3 Elementary bifurcations 217
- 11.4 Interactions of steady-state and Hopf bifurcations 222

- 11.5 Conclusion 234
- Acknowledgement 235
- References 235

12 Bifurcations and chaos in the system of Taylor vortices—laboratory and numerical experiment 238

V S L'vov, A A Predtechensky and A I Chernykh

- 12.1 Introduction 238
- 12.2 Formation of Taylor vortices 240
- 12.3 Power spectrum evolution 241
- 12.4 Breakdown of azimuthal wave coherence 243
- 12.5 Phenomenological model of the system of interacting Taylor vortices 246
- 12.6 Complicated behaviour of the system of interacting Taylor Vortices: numerical experiments 252
- 12.7 Statistical properties of trajectories in the phenomenological model 256
- 12.8 Statistical description of the system of interacting Taylor vortices in the direct interaction approximation 265
- 12.9 Evolution of the attractor structure with increasing Reynolds number 272
- 12.10 Concluding remarks 276
- Appendix 277
- Acknowledgements 278
- References 279

13 Hyperbolic attractors of differentiable dynamical systems 281

R V Plykin

- 13.1 Introduction 281
- 13.2 Preliminaries and statement of basic results 282
- 13.3 Theorem on invariant foliations 292
- Note on the proofs of the theorems 293
- Acknowledgements 293
- References 293

14 The intermittent transition to turbulence 295

Y Pomeau

- 14.1 Introduction 295
- 14.2 General description of intermittent transitions 296
- 14.3 Final remarks 302
- References 303

15 Resonance and bifurcation in Hopf-Landau dynamical systems 305*George R Sell*

- 15.1 Background 305
- 15.2 Normal singularity 307
- 15.3 Hopf-Landau dynamical systems 308
- 15.4 Open problems 311
 - Acknowledgements 312
 - References 312

16 Distinguishing deterministic and random systems 314*F Takens*

- 16.1 Introduction 314
- 16.2 Dynamical systems on compact metric spaces 321
- 16.3 Dynamical systems with an observable 325
- 16.4 Examples 327
 - References 332

17 Energy-conserving Galerkin approximations for the Bénard problem 334*Y M Treve*

- 17.1 Introduction 334
- 17.2 Equations of the Boussinesq model of Bénard convection 335
- 17.3 Energy-conserving Galerkin approximations 337
- 17.4 Properties of the $G^{(k)}$'s 339
- 17.5 Concluding remarks 341
 - Appendix 341
 - Acknowledgements 341
 - References 342

18 Stochasticity of the dynamical systems and the distribution of eigenvalues 343*G M Zaslavsky*

- 18.1 Introduction 343
- 18.2 Some definitions and assumptions 344
- 18.3 Historical observations 344
- 18.4 Statement of the problem 346
- 18.5 Billiard-type stochastic systems 347
- 18.6 Universality of systems with mixing 352
- 18.7 Basic results 353
- 18.8 Proof of the basic result 353
 - Acknowledgements 356
 - References 356