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# convection in Liquids



Springer-Verlag Berlin Heidelberg New York Tokyo

## **Convection in Liquids**

With 203 Figures

Springer-Verlag Berlin Heidelberg New York Tokyo 1984

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#### 172/2750 INSTITUT FÜR METEOROLOGIE U. KLIMATOLOGIP UNIVERSITÄT HANNOVER HERRENHÄUSER STR. 2 · 3000 HANNOVER 21

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